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BASIC LOGIC

DBM 20153 :

DISCRETE MATHEMATICS

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ABSTRACT

This eBook aims to enhance students' understanding of the topic of Boolean Algebra in Discrete Mathematics. The topic covered about Boolean Functions, Logic Gates and Karnaugh Map. Explanations of how to solve each tutorial question are shown in an interactive way. The difference between this eBook and other discrete math books is that its detailed explanations meant to serve as a reference to students as they master the concepts learned in the topic. In addition, every effort has been made to make this eBook sufficient for students' self study.

TABLE OF CONTENT

EXPLAIN PROPOSITIONAL LOGIC

EXPLAIN

COMPOUND PROPOSITION

1

·Compound proposition is the combination of two or more propositions.

2

·Proposition is a statement that is either true or false, but it cannot be both.

·The truth or falsity of a statement is called its truth value.

3

·A true statement has truth value T or 1, while a false statement has truth value F or 0.

4

·Propositions can be represented by using variables such as A, B, C, P, Q, R, etc.

5

·Statements which are questions, commands, expressions or opinions are not propositions.

EXAMPLE 1

Identify whether the following sentences are propositions or not. For a proposition, find its truth value and give a reason if it is not a proposition.

a) Perak is the biggest state in Peninsular Malaysia.

Answer : Proposition (True)

b) $2x^2 + 7x - 25 = 14;\ x = -1.$

Answer : Proposition (False)

c) Where is your hometown?

Answer : Not proposition because it is a question.

d) Fried chicken is the most delicous food in the world.

Answer : Not proposition because cannot determine the truth value since not everyone likes fried chicken.

e) Do not throw trash out of your vehicle.

Answer : Not proposition because it is a command.

f) $3 \in \{1, 3, 5, 7, 9\}$

Answer : Proposition (True)

g) Don't miss me too much!

Answer : Not proposition because it is an expression.

h) Hedgehog is an omnivore.

Answer : Proposition (True)

EXAMPLE 2

Identify whether the following sentences are propositions or not. For a proposition, find its truth value and give a reason if it is not a proposition.

a. The rainbow has seven colors.
Answer : Proposition (True)

b. Please ask if you do not understand.
Answer : Not proposition because it is a command.

c. 8 is a factor of 28.
Answer : Proposition (False)

d. $739 > 709$
Answer : Proposition (True)

e. There is life on mars.
Answer : Not proposition because the truth value is unpredictable

f. Tun Hussein Onn is the third Prime Minister of Malaysia.
Answer : Proposition (True)

g. I love you!
Answer : Not proposition because it is an expression.

h. These sentences are true.
Answer : Not proposition because the truth value is unpredictable.

i. How will you prove this argument?
Answer : Not proposition because it is a question.

j. Get me a glass of iced americano.
Answer : Not proposition because it is a command.

Identify whether the following sentences are propositions or not. For a proposition, find its truth value and give a reason if it is not a proposition.

k. Cristiano Ronaldo

Answer : Not proposition because it is just a name.

l. Crust is the earth's outermost surface.

Answer :Proposition (True)

m. $x^2 - 3x + 2 = 5$

Answer : Not proposition because x is unknown.

n. 32514 is an odd number.

Answer : Proposition (True)

o. $\frac{0}{0} = 1$

Answer : Proposition (False)

p. What is your name?

Answer : Not proposition because it is a question.

q. Kuala Lumpur is the capital city of Singapore

Answer : Proposition (False)

r. Human has 5 eyes

Answer : Proposition (False)

s. What a beautiful flower!

Answer : Not proposition because it is an expression.

t. Shut up!

Answer : Not proposition because it is a command.

LOGICAL CONNECTIVES

Negation (not, ~, ¬)

- Read as "not P".
- Turn a true proposition into false or a false proposition into true.
- Symbol: $\sim P$ or $\neg P$

P	$\sim P$
T	F
F	T

Conjunction (and, but, ∧)

- Read as "not P".
- Turn a true proposition into false or a false proposition into true.
- Symbol: $P \wedge Q$

Disjunction (or, ∨)

- Read as "P and Q" or "P but Q".
- The proposition is TRUE only when p and q are both true.
- Symbol: $P \vee Q$

P	Q	$P \wedge Q$	$P \vee Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Conditional Statement (If ... then ... , ⟶)

- Read as "If P then Q".
- The proposition is TRUE only when P and Q are both true and P is false (no matter what truth value Q has).
- Symbol : $P \rightarrow Q$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement (if and only if, ⟷)

- Read as "P if and only if Q".
- The proposition is TRUE when P and Q have the same truth values.
- Symbol: $P \leftrightarrow Q$

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Conditional Statement also, can be read as:

1. If P then Q
2. If P, Q
3. P is sufficient for Q
4. Q if P
5. Q when P
6. A necessary condition for P is Q
7. Q unless ~P
8. P implies Q
9. P only if Q
10. A sufficient condition for Q is P
11. Q whenever P
12. Q is necessary for P
13. Q follows from P

P is called hypothesis and Q is called the conclusion.

When the hypothesis and conclusion are identified in a conditional statement, three other statements can be derived:

Let : $P \rightarrow Q$

Converse $Q \rightarrow P$

Inverse $\sim P \rightarrow \sim Q$

Contrapositive $\sim Q \rightarrow \sim P$

WRITE PROPOSITION LOGIC IN ENGLISH

EXAMPLE 3

Let P: I am rich; and Q: I am happy. Write the following compound statements in symbolic forms.

a) I am rich and happy — Answer : $P \wedge Q$

b) I am poor but happy — Answer : $\sim P \wedge Q$

c) I am not rich or not happy — Answer : $\sim P \vee \sim Q$

d) I am happy if and only if I am not poor — Answer : $Q \leftrightarrow P$

e) It is not true that if I am poor, then I am not happy — Answer : $\sim(\sim P \rightarrow \sim Q)$

f) I am rich if I am happy — Answer : $Q \rightarrow P$

EXAMPLE 4

Let P : Men are immortal.
Q : Men are safe from tragedy.
R : Men are created by God
Express each of the following quantified formulas in English sentences.

a) $P \vee Q$

Answer : Men are immortal or men are safe from tragedy

b) $P \rightarrow (Q \wedge R)$

Answer : If men are immortal, then men are safe from tragedy and men are created by God

c) $\sim Q \rightarrow \sim P$

Answer : If men are not safe from tragedy, then men are not immortal

d) $\sim P \rightarrow (\sim Q \vee \sim R)$

Answer : If men are not immortal, then men are not safe from tragedy or men are not created by God

e) $(Q \wedge R) \leftrightarrow P$

Answer : Men are safe from tragedy and created by God if and only if men are immortal

EXAMPLE 5

Given P, Q and R are the propositions:

P : Amira will study Discrete Mathematics.

Q : Amira will go to the beach.

R : Amira is in a good mood.

Write the following compound statements in symbolic forms.

a Amira will not go to the beach and she will study the Discrete Mathematics.

Answer :

$\sim Q \wedge P$

b Amira does not study Discrete Mathematics if and only if Amira is not in a good mood.

Answer :

$\sim P \leftrightarrow \sim R$

c Amira will go to the beach if she is in a good mood.

Answer :

$R \rightarrow Q$

EXAMPLE 6

For each of the symbolic expression, write the corresponding (compound) statement base on the given primary statements.

P : A circle is a conic.
Q : is an irrational number.
R : Exponential series is convergent.

a $P \wedge \neg Q$

Answer : A circle is a conic and $\sqrt{5}$ is not an irrational number.

b $\neg P \vee Q$

Answer : A circle is not a conic or $\sqrt{5}$ is an irrational number.

c $P \rightarrow (Q \vee R)$

Answer : If a circle is a conic, then $\sqrt{5}$ is an irrational number or exponential series is convergent.

d $\neg P \leftrightarrow (Q \wedge \sim R)$

Answer : A circle is not a conic if and only if $\sqrt{5}$ s an irrational number and exponential series is not convergent.

Example 7: Write the converse, inverse and contrapositive statements for each of the following conditional statement

1 If it is sunny, then I will play baseball

Answer:
Converse :
If I will play baseball, then it is sunny.
Inverse :
If it is not sunny, then I will not play baseball.
Contrapositive :
If I will not play baseball, then it is not sunny.

2 If lines m and n are parallel, then lines m and n do not intersect

Answer:
Converse :
If lines m and n do not intersect, then lines m and n are parallel.
Inverse :
If lines m and n are not parallel, then lines m and n will intersect.
Contrapositive :
If lines m and n are intersecting, then lines and n are not parallel.

3 If Aaron did his homework, then he will pass this test

Answer:
Converse :
If Aaron pass this test, then he did his homework.
Inverse :
If Aaron did not do his homework, then he will fail this test.
Contrapositive :
If Aaron did not pass this test, then he did not do his homework.

4 If $x + 5 = 13$, then $x = 8$

Answer:
Converse :
If $x = 8$, then $x + 5 = 13$.
Inverse :
If $x + 5 \neq 13$, then $x \neq 8$.
Contrapositive :
If $x \neq 8$, then $x + 5 \neq 13$.

5 If an angle not measure 90°, then it is not a right angle

Answer:
Converse : If it is not a right angle, then an angle will not measure 90°.
Inverse : If an angle measures 90°, then it is a right angle.
Contrapositive : If it is a right angle, then an angle will measure 90°.

EXERCISE 1

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
a. Ipoh is the capital city of Selangor. g. Do not pass go.
b. Shah Alam is the capital city of Selangor. h. What time is it?
c. $2 + 3 = 5$ i. $4 + x = 5$
d. $5 + 7 = 10$ j. The moon is made of green cheese.
e. $x + 2 = 11$ k. $2n > 100$
f. Answer this question.

2. What is the negation of each of these propositions?
a. Today is Thursday.
b. There is no pollution in Kuala Lumpur.
c. $2 + 1 = 3$
d. The weather in Malaysia is hot and sunny.

3. Let p and q be the propositions
p: Swimming at the Port Dickson shore is allowed.
q: Sharks have been spotted near the shore.

Express each of these compound propositions as an English sentence.

a. $\sim q$
b. $p \wedge q$
c. $\sim p \vee q$
d. $p \rightarrow \sim q$
e. $\sim q \rightarrow p$
f. $\sim p \rightarrow \sim q$
g. $p \leftrightarrow \sim q$
h. $\sim p \wedge (p \vee \sim q)$

4. Let p and q be the propositions
p: It is below freezing.
q: It is snowing.

Write these propositions using p and q and logical connectives.
a. It is below freezing and snowing.
b. It is below freezing and but not snowing.
c. It is not below freezing and it is not snowing.
d. It is either snowing or below freezing (or both).
e. If it is below freezing, it is also snowing.
f. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
g. That it is below freezing is necessary and sufficient for it to be snowing.

5. Let p and q be the propositions
p: You drive over 65 miles per hour.
q: You get a speeding ticket.

Write these propositions using p and q and logical connectives.
a.You do not drive over 65 miles per hour.
b.You drive over 65 miles per hour, but you do not get a speeding ticket.
c.You will get a speeding ticket if you drive over 65 miles per hour.
d.If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
e.Driving over 65 miles per hour is sufficient for getting a speeding ticket.
f.You get a speeding ticket, but you do not drive over 65 miles per hour.
g.Whenever you get a speeding ticket, you are driving over 65 miles per hour.

6. Let p, q and r be the propositions
p: You have the flu.
q: You missed the final examination.
r: You pass the course.

Express each of these compound propositions as an English sentence.
a. $p \rightarrow q$
b. $\sim q \leftrightarrow r$
c. $q \rightarrow \sim r$
d. $p \vee q \vee r$
e. $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$
f. $(p \wedge q) \vee (\sim q \wedge r)$

7. Let p, q and r be the propositions
p: You get an A on the final exam.
q: You do every exercise in this book.
r: You get an A in this class.

Write these propositions using p, q and r and logical connectives.
a. You get an A in this class, but you do not do every exercise in this book.
b. You get an A on the final, you do every exercise in this book, and you get an A in this class.
c. To get an A in this class, it is necessary for you to get an A on the final.
d. You get an A on the final, but you don't do every exercise in this book, nevertheless, you get an A in this class.
e. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f. You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

8. Write the converse, inverse and contrapositive statements for each of the following conditional statement.
a. If two triangles are not similar, then their corresponding angles are not congruent.
b. If Ammara works hard, then she succeeds.
c. If the waves are small, I do not go surfing.
d. If x is an even integer, then (x + 1) is an odd integer.
e. If I am tall, then I will bump my head.

MORE NOTES AND EXERCISES VIA ONLINE

Explore these links to enhance your knowledge and understanding :

VIDEO

https://www.youtube.com/watch?v=hUMAyRcPZmo

https://www.youtube.com/watch?v=2UB9hMSAcl4

ONLINE ASSESSMENT

https://quizizz.com/join?gc=98555637

https://quizizz.com/join?gc=73879342

CONSTRUCT TRUTH TABLE

1 The truth value of a compound proposition built with logical connective is depends on the truth or falsify of its components.

2 Truth table is used to show the truth value of the compound proposition.

3 Tautology is a compound proposition that is "always true".

4 Contradiction is a compound proposition that is "always false".

5 ·Contingency is a compound proposition that is "neither contradiction nor tautology".

6 ·Recommended sequence of logical connectives in truth table:

1. Negation	~ or ¬
2. Bracket	( )
3. Conjunction	$\wedge$
4. Disjunction	$\vee$
5. Conditional/ Implication	$\rightarrow$
6. Biconditional	$\leftrightarrow$

EXAMPLE 8

Construct the truth table for each of the following compound propositions and determine whether it is tautology, contradiction or contingency

a) $(P \to Q) \vee (Q \to P)$

P	Q	$(P \to Q)$	$(Q \to P)$	$(P \to Q) \vee (Q \to P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$\therefore$ Tautology

b) $(p \leftrightarrow q) \wedge (\sim p \leftrightarrow q)$

p	q	$\sim p$	$(p \leftrightarrow q)$	$(\sim p \leftrightarrow q)$	$(p \leftrightarrow q) \wedge (\sim p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	F	T	F
F	F	T	T	F	F

$\therefore$ Contradiction

EXAMPLE 9

Construct the truth table for each of the following compound propositions and determine whether it is tautology, contradiction or contingency

a) $(P \rightarrow Q) \vee (Q \rightarrow R)$

P	Q	R	$(P \rightarrow Q)$	$(Q \rightarrow R)$	$(P \rightarrow Q) \vee (Q \rightarrow R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

∴ Tautology

b) $(p \wedge \sim q) \rightarrow \sim r$

p	q	r	$\sim q$	$\sim r$		$(p \wedge \sim q)$	$(p \wedge \sim q) \rightarrow \sim r$
T	T	T	F	F		F	T
T	T	F	F	T		F	T
T	F	T	T	F		T	F
T	F	F	T	T		T	T
F	T	T	F	F		F	T
F	T	F	F	T		F	T
F	F	T	T	F		F	T
F	F	F	T	T		F	T

Contingency

EXAMPLE 10

Construct the truth table for each of the following compound propositions and determine whether it is tautology, contradiction or contingency

a) $(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$(p \vee \sim q)$	$(p \wedge q)$	$(p \vee \sim q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

∴ Contingency

b) $(P \vee \sim Q) \leftrightarrow (Q \wedge \sim P)$

P	Q	$\sim P$	$\sim Q$	$(P \vee \sim Q)$	$(Q \wedge \sim P)$	$(P \vee \sim Q) \leftrightarrow (Q \wedge \sim P)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	F	F

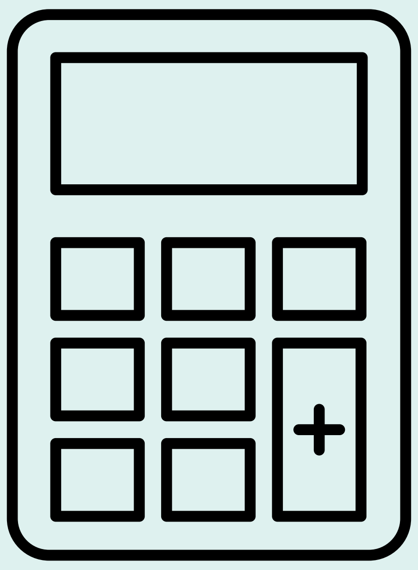
∴ Contradiction

c) $(p \rightarrow q) \vee (\sim p \rightarrow r)$

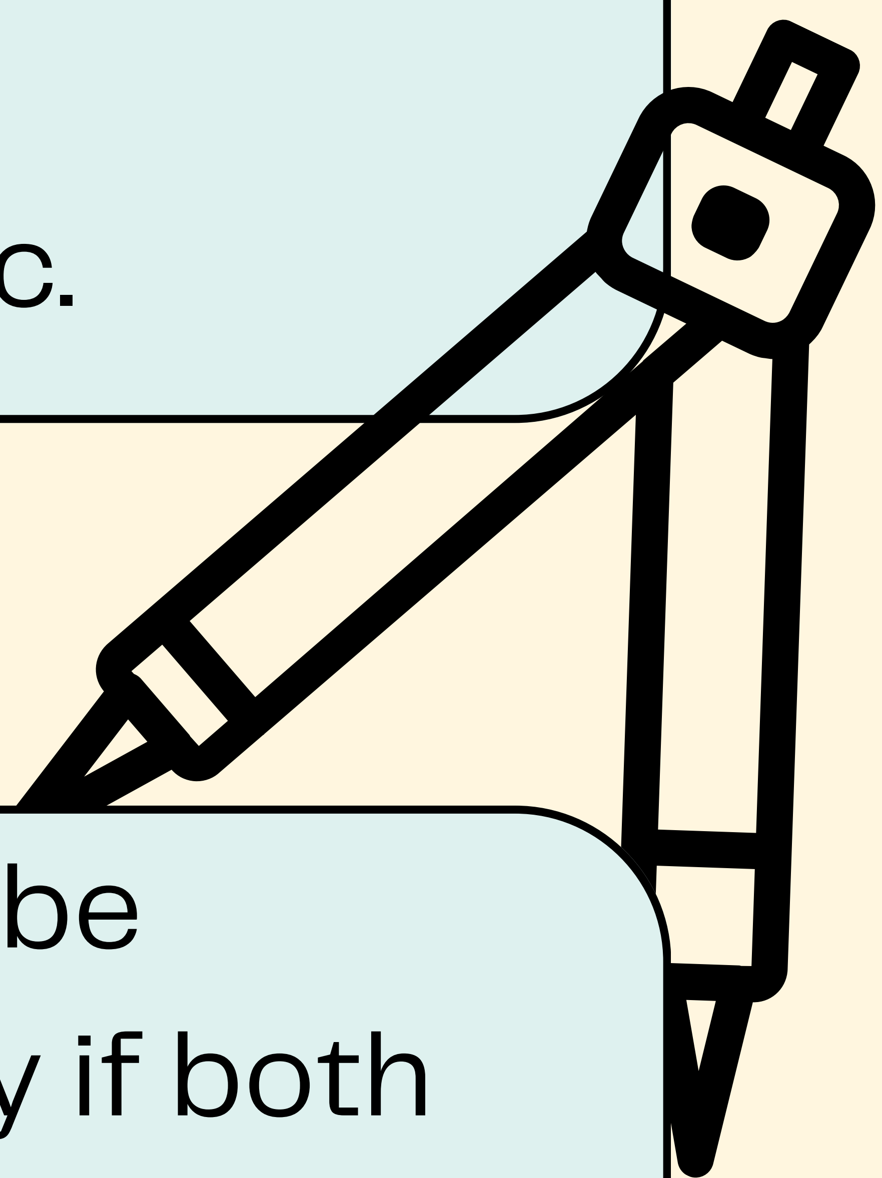
p	q	r	$\sim p$	$(p \rightarrow q)$	$(\sim p \rightarrow r)$	$(p \rightarrow q) \vee (\sim p \rightarrow r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

∴ Tautology

Logical Equivalence

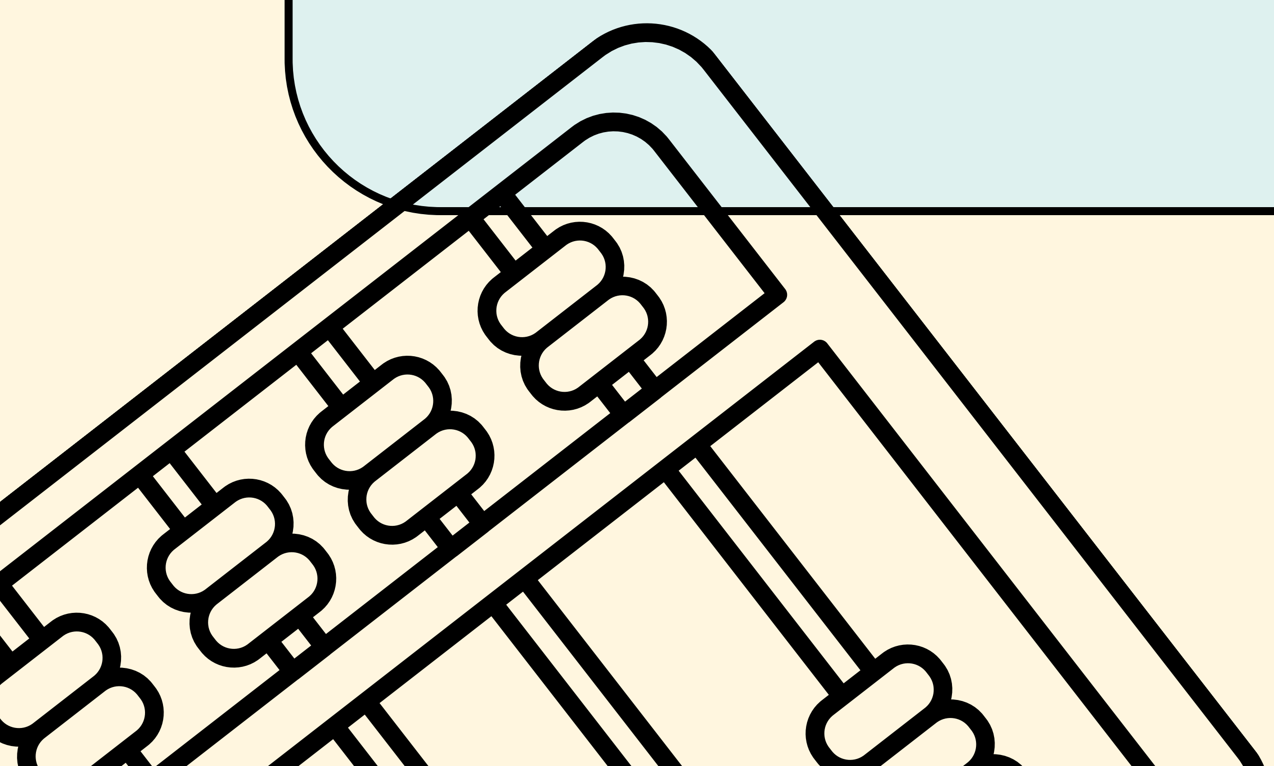


1 Logical equivalence is one of the features of propositional logic.



2 Two propositions are said to be logically equivalent if and only if both columns in the truth table are identical to each other.

3 The notation $P \Leftrightarrow Q$ or $P \equiv Q$ denotes that P and Q are logically equivalent.



EXAMPLE 11

a) Show that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent

p	q	$(p \vee q)$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

∴ Logically equivalent

b) Show that $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

∴ Logically equivalent

EXAMPLE 12

Construct the truth table for each of the following compound propositions and determine whether it is logically equivalent or not.

a) $\sim(p \leftrightarrow q)$ and $p \leftrightarrow \sim q$

p	q	$\sim q$	$(p \leftrightarrow q)$	$\sim(p \leftrightarrow q)$	$p \leftrightarrow \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	F

∴ Logically equivalent

b) $\sim r \wedge (p \rightarrow q)$ and $(\sim p \leftrightarrow \sim q) \vee r$

p	q	r	$\sim r$	$p \rightarrow q$	$\sim r \wedge (p \rightarrow q)$	$\sim p$	$\sim q$	$\sim p \leftrightarrow \sim q$	$(\sim p \leftrightarrow \sim q) \vee r$
T	T	T	F	T	F	F	F	T	T
T	T	F	T	T	T	F	F	T	T
T	F	T	F	F	F	F	T	F	T
T	F	F	T	F	F	F	T	F	F
F	T	T	F	T	F	T	F	F	T
F	T	F	T	T	T	T	F	F	F
F	F	T	F	T	F	T	T	T	T
F	F	F	T	T	T	T	T	T	T

∴ Not logically equivalent

c) $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

p	q	r	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \vee (q \rightarrow r)$	$(p \wedge q)$	$(p \wedge q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

∴ Logically equivalent

EXERCISE 2

1. **Construct the truth table of the following compound propositions.**
   a. $(p \vee q) \leftrightarrow (q \vee p)$
   b. $\sim p \vee q$
   c. $\sim(\sim p \wedge q)$
   d. $(p \leftrightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
   e. $(p \rightarrow r) \leftrightarrow (q \wedge \sim p)$
   f. $\sim(p \vee q) \rightarrow (r \vee \sim q)$
   g. $(p \vee q) \wedge \sim r$
   h. $(p \leftrightarrow q) \vee (\sim q \leftrightarrow r)$

2. **Construct the truth table for each of the following compound propositions and determine whether it is tautology or not.**
   a. $\sim p \wedge (p \rightarrow q) \rightarrow \sim q$
   b. $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$
   c. $(p \wedge q) \rightarrow p$
   d. $\sim p \rightarrow (p \rightarrow q)$
   e. $\sim(p \rightarrow q) \rightarrow p$
   f. $(p \wedge q) \rightarrow (p \rightarrow q)$
   g. $\sim(p \rightarrow q) \rightarrow \sim q$
   h. $[\sim p \wedge (p \vee q)] \rightarrow q$
   i. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
   j. $[p \wedge (p \rightarrow q)] \rightarrow q$
   k. $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
   l. $[\sim p \wedge (p \rightarrow q)] \rightarrow \sim q$
   m. $[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$

3. **Construct the truth table for each of the following compound propositions and determine whether it is logically equivalent or not.**
   a. $(p \leftrightarrow q)$ & $(p \wedge q) \vee (\sim p \wedge \sim q)$
   b. $(p \rightarrow q) \wedge (p \rightarrow r)$ & $p \rightarrow (q \wedge r)$
   c. $(p \rightarrow r) \wedge (q \rightarrow r)$ & $(p \vee q) \rightarrow r$
   d. $(p \rightarrow q) \vee (p \rightarrow r)$ & $p \rightarrow (q \vee r)$
   e. $\sim p \rightarrow (q \rightarrow r)$ & $q \rightarrow (p \vee r)$
   f. $p \leftrightarrow q$ & $(p \rightarrow q) \wedge (q \rightarrow p)$
   g. $p \leftrightarrow q$ & $\sim p \leftrightarrow \sim q$
   h. $(p \rightarrow q \rightarrow r$ & $p \rightarrow (q \rightarrow r)$
   i. $(p \wedge q) \rightarrow r$ & $(p \rightarrow r) \wedge (q \rightarrow r)$

MORE NOTES
AND EXERCISES VIA ONLINE

Explore these links to enhance your knowledge and understanding :

VIDEO

https://www.youtube.com/watch?v=XwC2L_zu-DM

ONLINE ASSESSMENT

https://quizizz.com/join?gc=83736291

https://quizizz.com/join?gc=73441462

PREDICATE LOGIC

01.

COMPOUND STATEMENT IS THE COMBINATION OF STATEMENTS BY USING LOGICAL CONNECTIVES.

02.

PREDICATE IS AN EXPRESSION OR A VERB PHRASE TEMPLATE THAT DESCRIBES A PROPERTY OF OBJECTS OR A RELATIONSHIP AMONG OBJECTS REPRESENTED BY ONE OR MORE VARIABLES.

03. **FOR EXAMPLES:**

- Let $E(x, y)$ denote "$x = y$"
- Let $X(a, b, c)$ denote "$a + b + c = 0$"
- Let $M(x, y)$ denote "x is married to y"

EXAMPLE 13

State the expression of predicate in the following statements:

a

The car, Karim is driving is blue.
The sky is blue.
The cover of this book is blue.

Answer :
Blue (car)
Blue (sky)
Blue (cover of book)
∴ B (x)

b

Ali gives the book to Mariam.
Karim gives a loaf of bread to Abu.
Aminah gives a lecture to Mariam.

Answer :
Gives (Ali, book, Mariam)
Gives (Karim, bread, Abu)
Gives (Aminah, lecture, Mariam)
∴ G (x, y, z)

QUANTIFIERS

01

·The variable of predicates is quantified by quantifiers.

·Predicates are used alongside quantifiers to express the extent to which a predicate is true over a range of elements.

02

03

·There are two types of quantifier:

Universal, ∀	Existential, ∃
• Mathematical statements sometimes state that **a property** is **true** ***FOR ALL*** **the values of a variable** in a particular domain, called the domain of discourse. • Read as: o All or Every… (for object) o Everyone … (for people)	• Some mathematical statements state that ***THERE EXISTS*** **at least one element with a certain property**. • Read as: o Some … (for object) o Someone … (for people)

01

·Statements which contain quantifier words such as "all", "none", "some" or "there exists" are called quantified statements.

·When reading quantified statements in English, read them from left to right.

·For example, let the universe of discourse be the set of cars and the predicate F (x, y) denote as "x is faster than y".

$\forall x \, \forall y \, F(x, y)$	All car is faster than all car.
$\forall x \, \exists y \, F(x, y)$	All car is faster than some cars.
$\exists x \, \forall y \, F(x, y)$	Some cars are faster than all car.
$\exists x \, \exists y \, F(x, y)$	Some cars are faster than some cars.

EXAMPLE 14

Let E (x) = x is even and G (x, y) = x > y as the universe of discourse be the set of natural numbers. Write the following predicate logic to English sentences and vice versa.

a. $\forall x\, \exists y\, G(x, y)$

Answer : Every number is greater than some numbers.

b. $\exists x\, \forall y\, G(x, y)$

Answer :Some numbers are greater than every number.

c. $\exists x\, \sim E(x)$

Answer : Some numbers are not even.

d. $\forall x\, E(x)$

Answer : Every number is even.

e. 6 is an even number.

Answer : $E(6)$

f. Some numbers are greater than some numbers.

Answer : $\exists x\, \exists y\, G(x, y)$

g. Some numbers are greater than 10.

Answer : $\exists x\, G(x, 10)$

EXAMPLE 15

Let T (x, y) = x is taller than y as the universe of discourse be the set of people. Write the following predicate logic to English sentences and vice versa.

a $\forall x\, \exists y\, T(x, y)$

Answer : Everyone is taller than someone.

b $\exists x\, \forall y\, T(x, y)$

Answer : Someone is taller than everyone.

c $\exists x\, \forall y\, {\sim}T(x, y)$

Answer : Someone is not taller than everyone.

d $\exists x\, \forall y\, {\sim}T(x, y)$

Answer : Not everyone is taller than someone.

e Someone is not taller than someone.

Answer : $\exists x\, \exists y\, {\sim}T(x, y)$

f Alia is taller than Amin.

Answer : T(Alia, Amin)

g Someone is taller than Rashid.

Answer : $\exists x\, T(x, Rashid)$

EXAMPLE 16

1.Let H (x) be the statement "x is happy", where the universe of discourse for x is the set of people. Express each of the following quantifications in English.

a

$\exists x \, H(x)$

Answer : Someone is happy.

b

$\sim\forall x \sim H(x)$

Answer : Not everyone is not happy.

c

$\exists x \sim H(x)$

Answer : Someone is not happy.

d

$\forall x \sim H(x)$

Answer : Everyone is not happy.

1. Let P (x) be the statement "x likes reading" and Q (x) be the predicate "x can speak French". The domain for both predicates are lecturers in PBU. Use quantifiers and logical connectives to express each of the following statements.

a) Some lecturers in PBU like reading and can speak French.

Answer : $\exists x\,[P(x) \wedge Q(x)]$

b) Every lecturer in PBU likes reading if they cannot speak French

Answer : $\forall x\,[\sim Q(x) \rightarrow P(x)]$

c) Someone can speak French.

Answer : $\exists x\, Q(x)$

d) Everyone likes reading or can speak French.

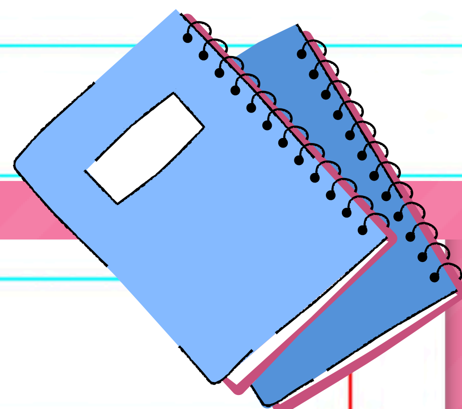
Answer : $\forall x\,[P(x) \vee Q(x)]$

e) If everyone likes to read, then someone can speak French.

Answer : $\forall x\, P(x) \rightarrow \exists x Q(x)$

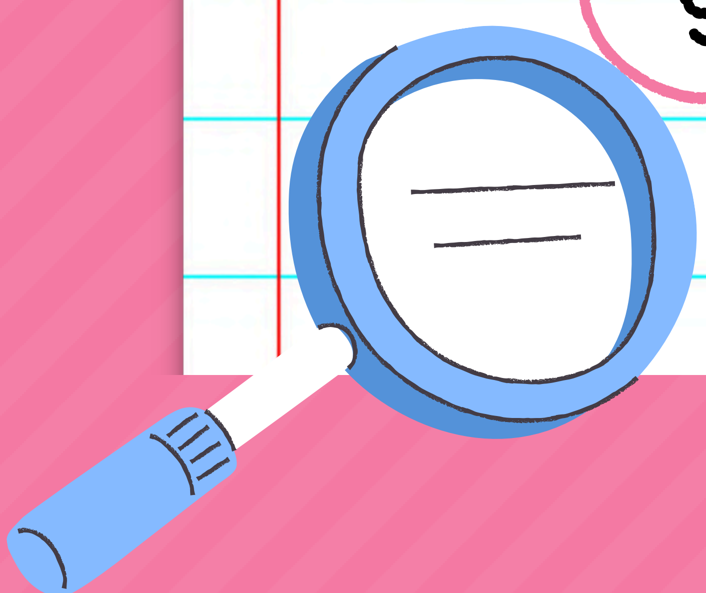
f) Sabrina likes to read.

Answer : P (Sabrina)



g) Ali cannot speak French.

Answer : $\sim Q(Ali)$



EXERCISE 3

. Assume $P(x, y)$ is the predicate of "x is prettier than y", and let the universe of discourse be the set of branded shoes. Use quantifiers to express each of the following statements.

a. Not all branded shoes are prettier than all branded shoes.
b. Some branded shoes are prettier than every branded shoe.
c. Some branded shoes are prettier than Bonia shoes.
d. Some branded shoes are not prettier than some branded shoes.

. Complete these specifications into English where $F(x)$ is "x is out of service", $B(x)$ is "x is busy", $L(y)$ is "y is lost" and $Q(y)$ is "y is queued". The domain of x is all printers and the domain for y is all printer jobs.

a. $\forall x\, B(x) \leftrightarrow \exists y\, Q(y)$
b. $\exists y\, [Q(y) \wedge L(y)] \rightarrow \neg\forall x\, F(x)$
c. $\forall x\, B(x) \vee [\forall y\, Q(y) \rightarrow \exists y\, L(y)]$
d. $\forall y\, [\neg L(y) \vee \neg Q(y)] \leftrightarrow \forall x\, [\neg F(x) \wedge \neg B(x)]$

. Let $M(x, y)$ be the statement "x has sent an email to y", where the universe of discourse consists of all students in your class. Use quantifiers to express each of the following statements.

a. Chen has never sent an email to Jenny.
b. Every student in your class has sent an email to Sarah.
c. There is a student in your class who sent an email to everyone in your class.
d. Every student in your class has sent an email to some students in the class.

MORE NOTES AND EXERCISES VIA ONLINE

Explore these links to enhance your knowledge and understanding :

VIDEO

https://www.youtube.com/watch?v=VPn7ArmFNYA

ONLINE ASSESSMENT

https://quizizz.com/join?gc=88042202

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